## R (and RStudio) for Econometrics

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## Which one?

- People often ask which computer language should I use?
- But we're economists, right? So we use.... Stata? Eviews? Matlab? Gauss? Ox? C++? Fortran? Python? Julia? (Say it quietly.... Excel?)
- These are all fine, so why another? Why specifically R?
- It's up to you to decide, but for me an engaged user community is the answer. And it's free. Free is good.


## R and the R GUI

- $R$ is a scripting language designed to be suited to statistical analysis
- Particularly good at handling data
- Built to be a super calculator that does all the maths we need
- Excellent graphical and tabular output
- Multiplatform: runs on Windows, Mac, Linux and hardware as humble as the Raspberry Pi
- Free to use - simply download from https://cran.r-project.org/
- Supported by an extraordinary user community
- https://www.r-bloggers.com/
- https://stackoverflow.com/
- \#rstats
- Most of these resources are aimed at data scientists


## Learning econometrics

- Typical intermediate textbooks are a good way to start, and Wooldridge (2019) leads the pack. We can use it as a template to understand how to use $R$
- One learning strategy is to do the example and exercises in Wooldridge (2019), which needs data
- Data is available in an R package - wooldridge , see Shea (2021)
- Code is explicitly in Heiss (2020) and available from the associated website
- Texts covering specific using R are appearing:
- Adams (2021) for microeconometrics
- Cunningham (2021) for causal methods
- Huntington-Klein (2022) as an elementary text for data analysis and research design that should appeal to the budding econometrician


## How to use R

- $R$ is designed for data manipulation and analysis, often associated with data science rather than econometrics
- Data science skills are covered in Freeman and Ross (2019), and many of them are useful for us too
- Lots of helpful books to help with these sorts of skills in R - too many to mention even a fraction


## Workflow

We should approach econometric analysis in this stylized way:

- Read, manipulate, clean, visualize in core/specialized packages
- Do econometric analysis either:
- Using appropriate libraries, either part of the core packages (lm) or from specialist packages (say ivreg)
- Or write native R code to do the analysis
- Display output, graphs etc using core/specialist R routines

The first and last of these are common to any statistical/modelling analysis, the middle bit is where the econometrician is doing something different

## Basic GUI

- $R$ by itself comes with a simple but comprehensive interface:



## RStudio

- Most people use a more comprehensive development environment such as RStudio from posit. After you download R, I would encourage you to download and install this as your GUI. It looks a lot like, say, Matlab.
- Rstudio looks something like:

- Left side as shown contains R itself
- Right side displays a lot of information:
- Top panel, first two tabs

- It is very configurable, and supports lots of languages other than R. Note by default it uses a 'black-on-white' text colour scheme but offers any number of variations. Choose one that suits your eyes, particularly if you spend a lot of time in front of the screen!


## Simple first example

- Use R to create a matrix:


```
A <- matrix(c(1, 2, 3,4), 2, 2)
```

A
\#\# [,1] [,2]
\#\# [1,] 1
\#\# [2,] 24


Seems easy enough: try and figure out what the function matrix( ) is doing. Change the values to all 4 s , say.

## Econometrics is linear algebra

That's a bit extreme, but you mostly need to do linear algebra to program up many of the estimators we use. If we wanted to program an estimator we need to know a bit more.

## Some more linear algebra

Assume the following: $A$ and $B$ are real matrices of dimension $n \times n, b$ and $c$ are real $n$-vectors, $X$ is a real $T \times k$ matrix, and $S$ is a symmetric real matrix.

Maths commands essential to linear algebra
Maths

| Hadamard product | $A \odot B$ | A * B | Element-by-element, $A, B$ same size |
| :---: | :---: | :---: | :---: |
| Matrix/vector product | $A \times B, A \times b$ | A \% $\%$ \% B, A \% $\%$ b | Normal product rule |
| Inner product | $X^{\prime} X$ | $t(X) \% * \% ~ X ~$ | Also uses transpose operator, t() |
|  |  | crossprod(X) | More efficient, but less mathy |
|  | $A^{\prime} B$ | t (A) \%*\% B |  |
|  |  | crossprod ( $A, B$ ) |  |
| Outer product | $A \times B^{\prime}$ | tcrossprod (A, $\mathrm{B}^{\text {) }}$ |  |
| Inverse | $A^{-1}$ | solve(A) | Matrix inverse is a special case of... |
| Solve for $d$ | $A d=b \Rightarrow d=A^{-1} b$ | d <- solve(A, b) | ...linear solution! |
| Cholesky decomp | $S=R^{\prime} R$ | R <- chol(S) | $S$ is a symmetric, positive definite matrix |
| Cholesky inverse | $S^{-1}$ | chol2inv(chol(S)) | Fast! |


| Determinant |  | $\operatorname{det}(\mathrm{A})$ |  |
| :--- | :--- | :--- | :--- |
| Diagonal | $\operatorname{diag}(\mathrm{A})$ | Retrieve the elements $a_{i i}$, <br> $i=1, \ldots, n$ |  |
| of a matrix | $\mathrm{A}<-\operatorname{diag}(\mathrm{b})$ | Set the diagonal of $A$ to $b$, <br> zero elsewhere |  |
| in a matrix | $\operatorname{diag}(\mathrm{n})$ | Returns a list: $\mathrm{E} \$ \mathrm{values}$, <br> Identity matrix | $I_{n}$ |

## Programming example: Simple linear algebra

Consider the following simultaneous system of equations:

$$
\begin{aligned}
x_{1}+2 x_{2} & =6 \\
x_{1}-3 x_{2}+2 x_{3} & =0 \\
-2 x_{1}+3 x_{3} & =2
\end{aligned}
$$

Find the values of $x$ that solve this using R .
Hint - write the problem in matrix form

$$
A x=b
$$

where

$$
A=\left[\begin{array}{ccc}
1 & 2 & 0 \\
1 & -3 & 2 \\
-2 & 0 & 3
\end{array}\right], \quad b=\left[\begin{array}{l}
6 \\
0 \\
2
\end{array}\right]
$$

and then use solve.

## R solution

$R$ code to create these matrices is:

```
A <- matrix(c(1,1,-2,2,-3,0,0,2,3),3,3) # Matrices are populated by column by default
b <- matrix(c(6,0,2),3,1)
```

The solution is:

```
x <- solve(A,b)
```

where x is:

```
## [,1]
## [1,] 2
## [2,] 2
## [3,] 2
```

(Check it by eye!)

## Precision

Take a real matrix $A_{m n}$ with $n \leq m$ and pre-multiply by its own transpose, i.e. $B=A^{\prime} A$. $B$ is then symmetric, positive semi-definite. If $\operatorname{rank}(A)=n$, then $\operatorname{rank}(B)=n$ and positive definite, and its inverse exists.

```
## [,1] [,2]
## [1,] 1.04 0.2
## [2,] 0.20 1.0
```

Let's invert $B$ three different ways and premultiply the answers by $A$.

```
i1 <- solve(B)
i2 <- chol2inv(chol(B))
i3 <- qr.solve(B)
i1 %*% B
```

| \#\# | $[, 1]$ | $[, 2]$ |
| :--- | ---: | ---: |
| \#\# [1, ] | $1.000000 \mathrm{e}+00$ | 0 |
| \#\# [2, ] $2.775558 \mathrm{e}-17$ | 1 |  |

i2 \%* B

| \#\# | $[, 1]$ | $[, 2]$ |
| :--- | ---: | ---: |
| $\# \#$ | $[1]$, | 1 |
| $5.551115 e-17$ |  |  |

```
## [2,]
```

$01.000000 \mathrm{e}+00$

```
i3 %*% B
```

| \#\# | $[, 1]$ | $[, 2]$ |
| :--- | ---: | ---: |
| \#\# $[1]$, | 1 | $-2.775558 \mathrm{e}-17$ |
| \#\# $[2]$, | 0 | $1.000000 \mathrm{e}+00$ |

We see some rounding differences. This is a fundamental characteristic of numerical linear algebra.

## Programming the regression problem

Let's look at the familiar regression problem for some generated data.

$$
y=X b+\epsilon
$$

where $\epsilon \sim N(0, .2), X$ is a $(k+1) \times n$ matrix of regressors including a constant and $b$ a $k+1$ vector of coefficients. Let's generate some random data of an arbitrary sized problem:

```
X <- matrix(rnorm(180, 2, 1), 60, 3)
head(X, 6) # Print first six rows
```

| \#\# | $[, 1]$ | $[, 2]$ | $[, 3]$ |
| :--- | ---: | ---: | ---: |
| \#\# $[1]$, | 2.409870 | 0.7925486 | 2.749149 |
| \#\# [2, ] | 1.710207 | 2.7867654 | 3.261341 |
| \#\# [3,] | 3.525394 | 2.9972849 | 1.951609 |

```
## [4,] 2.966957 1.8696408 2.667315
## [5,] 1.538812 1.8504980 1.146760
## [6,] 1.640619 3.4909867 1.411701
```

```
X <- cbind(1, X) # Add a constant
tail(X,6) # Print last six rows
```

| \#\# | $[, 1]$ | $[, 2]$ | $[, 3]$ | $[, 4]$ |
| :--- | ---: | ---: | ---: | ---: |
| \#\# $[55]$, | 1 | 2.2649989 | 2.6552562 | 0.8168622 |
| \#\# $[56]$, | 1 | 0.2889892 | 0.2898367 | 2.2295481 |
| \#\# $[57]$, | 1 | 2.5199898 | 2.6010970 | 3.7696509 |
| \#\# [58,] | 1 | 1.2229275 | 1.0048883 | 2.0531373 |
| \#\# [59,] | 1 | 1.0409328 | 2.0610489 | 2.4206702 |
| \#\# [60,] | 1 | 2.3584212 | 0.9789181 | 1.4413930 |

Now create a dependent variable that is a linear combination of these variables plus some noise. Create the linear relationship first so we know what it is:

```
b <- matrix(c(0.5,1,-1,.2), 4, 1)
```

and then the dependent variable:

```
y<- X %*% b + 0.2*rnorm(60)
```

We could now do a regression - i.e. calculate

$$
\hat{b}=\left(X^{\prime} X\right)^{-1} X^{\prime} y
$$

which can be written:

```
bhat <- solve(t(X)%*%X)%*%t(X)%*%y
```

which gives

```
## [,1]
## [1,] 0.5376433
## [2,] 0.9776018
## [3,] -0.9828396
## [4,] 0.1816978
```

But I wouldn't do it like this (we'll see why in a minute). A better way would be

```
bhat2 <- chol2inv(chol(crossprod(X)))%*%crossprod}(\textrm{X},\textrm{y}
```

or even

```
bhat3 <- qr.solve(X,y)
```

which both evaluate to the same $\hat{b}$ values.

## Testing timings

Why does it matter how you do things? It should be obvious that it might, but it turns out some fairly trivial things can make a lot of difference. We set some parameters so we can create a bigger problem.

We will use seven different methods to calculate an estimate of $b$. These are two variations on the three calculations below (where the brackets matter!):

$$
\begin{gathered}
\hat{b}_{1}=\left(\left(X^{\prime} X\right)^{-1}\right) X^{\prime} y \\
\hat{b}_{2}=\left(\left(X^{\prime} X\right)^{-1}\right)\left(X^{\prime} y\right) \\
\hat{b}_{3}=\left(\left(X^{\prime} X\right)^{-1}\left(X^{\prime} y\right)\right)
\end{gathered}
$$

where we do it either 'by hand' or using crossprod, plus using qr. solve.
The timings for these different methods are:

Timings of different numerical regression methods

```
qr.solve(X,y)
chol2inv(chol(crossprod(X))) %*% crossprod(X,y)
solve(crossprod(X), crossprod(X,y))
solve(crossprod(X)) %*% crossprod(X,y)
solve(t(X)%*%X, (t(X)%*%y))
solve(t(X)%*%X) %*% (t(X)%*%y)
solve(t(X)%*%X) %*% t(X) %*% y
0
5
10
1 5
Seconds taken to do 10000 replications
```


## Using lm instead

- Instead of all this we could use the built in regression command lm

```
library(tidyverse)
data <- as_tibble(cbind(y0, X0)) %>%
    rename_all(~ c("y", "c", "X1", "X2", "X3"))
```

```
reg1 <- lm(y ~ X1 + X2 + X3, data = data)
summary(reg1)
```

```
##
## Call:
## lm(formula = y ~ X1 + X2 + X3, data = data)
##
## Residuals:
\#\# Min 1Q Median 3Q Max
## -0.35502 -0.10460 -0.00169 0.09132 0.30085
##
## Coefficients:
## Estimate Std. Error t value Pr(>|t|)
\#\# (Intercept) 0.53764 0.07489 7.179 1.75e-09 ***
\#\# X1 0.97760 0.01992 \(49.087<2 e-16^{* * *}\)
\#\# X2 -0.98284 0.01764-55.711 < 2e-16 ***
\#\# X3 0.18170 0.02403 7.561 4.09e-10 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1612 on 56 degrees of freedom
## Multiple R-squared: 0.9886, Adjusted R-squared: 0.988
## F-statistic: }1617\mathrm{ on 3 and 56 DF, p-value: < 2.2e-16
```

- This does all the work for us, but if the estimator doesn't exist, you need to do it yourself!
- Notice I load the tidyverse
- We need to understand every part of this


## tidyverse

Essentially two ways to use R: with the tidyverse or without it. I leave it to you to decide, because although opinion is split, sometimes people have practical rather than purist responses:

## Andie Perreault

@SineAndie

> Slowly converting from base $R$ to the tidyverse and sweet mother of God, I just mean standardized by age and survey for a subset of years AND plotted it all in SIX SHORT LINES OF CODE. I am forever in debt to the \#tidyverse wizards

3:21 PM - 22 May 2019

This is a collection of libraries that all work together to (amongst othere things) manipulate and plot data. For a description see Wickham et al. (2019).

I use elements of the tidyverse throughout any code I write: in particular I often use commands from the following libraries for data wrangling and plotting:

## - dplyr

- select - retain/drop columns
- filter - conditionally choose rows
- slice - retain/drop rows by position
mutate - create a new variable
rename - rename an old variable
- tidyr
- pivot_longer - make wide data long
- pivot_wider - make long data wide
- broom
- tidy - the Ronseal of the tidyverse
- lubridate
- year - this returns the year from a date
- magrittr
- \%>\% - a pipe operator that chains together commands to make manipulating data more understandable and easier to program
- ggplot2
- ggplot - initiate a graph
- geom_line - draw a line etc.

For the latter Wickham (2016) is the main reference, but learning by doing is the only way.

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## Appendix

## Some programming basics

A few not-linear-algebra commonly used commands used are summarized in the following table:

|  | R | Example | Notes |
| :--- | :--- | :--- | :--- |
| Assign a value | $<-$ | $\mathrm{a}<-4$ | Also legal is $a=4$. <br> But $I$ hate it. |
| Create $a$ list of <br> values | $c()$. | $v<-c(1,-2,22)$ | Defining 'on the fly' |


|  | R | Example | Notes |
| :--- | :--- | :--- | :--- |
| Sequence | seq(i, k, l) | $5,7, \ldots, 21$ | Create a sequence |

## Functions

Everything in $R$ is a function (although it doesn't look like it). Defining a function is simple:

```
name_of_function <- function(function_arguments){
    # Body of function where stuff is done
}
```

Here's one that actually does something:

```
threetimesadd <- function(x,y){
    z<- 3*(x+y)
```

and if we run it:
threetimesadd $(4,6)$
\#\# [1] 30

